Error Correction in Vector Network Analyzers

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Abstract

This article describes systematic errors encountered in vector network analysis and how they can be mathematically corrected. The focus lies on the application to a unidirectional VNA, i.e. where the 2-port device needs to be manually turned around to measure the full 2-port S-matrix.

1 What does a VNA measure?

Figure 1 shows a very general and abstract model of a unidirectional vector network analyzer (VNA).

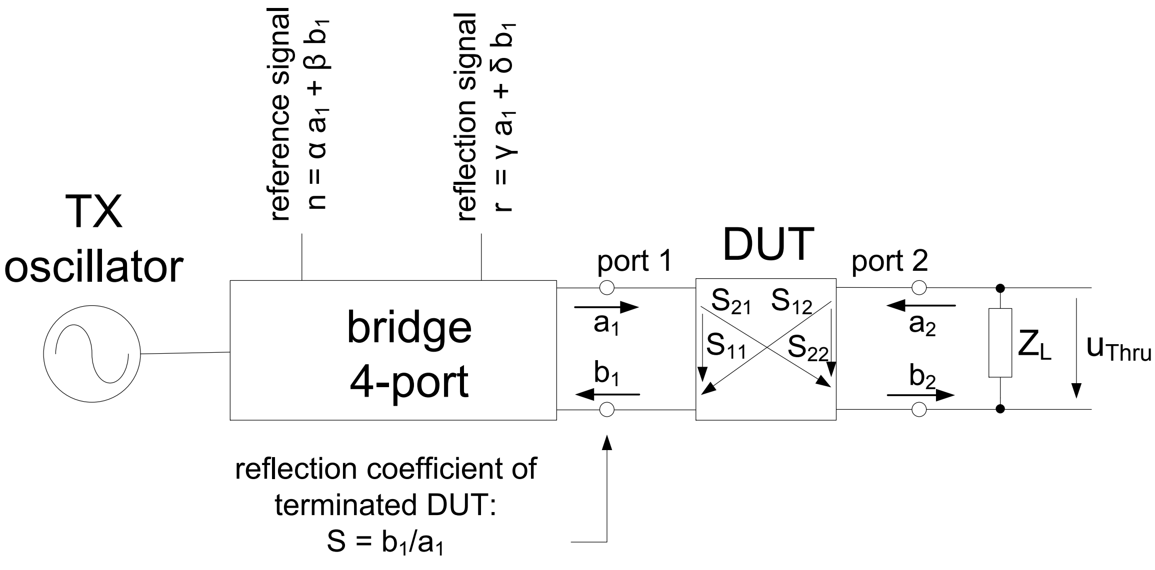

Figure 1: Abstract model of a unidirectional VNA.

The signal power of a TX oscillator is fed into a reflection bridge or directional coupler, which can be described as a linear 4-port device. Port 1 is connected to the oscillator, port 2 to the DUT, port 3 is the measured reflect signal r and port 4 is the measured reference signal n. Observe the wave amplitude a_1 travelling from the bridge to the DUT. This wave is partially reflected by the DUT leading to the wave amplitude b_1.

1.1 Measuring 1-port reflection coefficients

The reflection coefficient S of the DUT input with its output terminated by Z_L can be determined from these wave amplitudes by equation 1.

$$S = \frac{b_1}{a_1} \tag{1}$$

An ideal VNA would measure a_1 as reference signal and b_1 as reflect signal and calculate the reflection coefficient S according to equation 1. Since the reflection bridge shown in figure 1 is a linear 4-port device, the signals at any port can always be described as a linear combination of the wave amplitudes a_1 and b_1. Thus, the reflect and reference signals are of the following form:

$$r = \gamma a_1 + \delta b_1 \tag{2}$$

$$n = \alpha a_1 + \beta b_1 \tag{3}$$

And the measurement result M is:

$$M = \frac{r}{n} = \frac{\gamma a_1 + \delta b_1}{\alpha a_1 + \beta b_1} \tag{4}$$

By virtue of equation 1 we obtain the following result:

$$M = \frac{\gamma + \delta S}{\alpha + \beta S} \tag{5}$$

This function is called a Moebius [1] transform. It actually contains only 3 independent parameters as one of $\{\alpha, \beta, \gamma, \delta\}$ can be divided out. Thus, without loss of generality, M can be written in the following way:

$$M = \frac{S + a}{bS + c} \tag{6}$$

If the three parameters $\{a, b, c\}$ are known, the input reflection coefficient S of the DUT can be calculated from the measurement result M by inverting equation 6:

$$S = \frac{a - cM}{bM - 1} \tag{7}$$

The determination of the parameters $\{a, b, c\}$ is quite simple. It is just necessary to measure three calibration standards with well known reflection coefficients, e.g. $\{S_O, S_S, S_L\}$. Usually, but not necessarily, an open a short and a load calibration standard are used. Measuring these, one obtains the measurement results $\{M_O, M_S, M_L\}$, where by virtue of equation 6:

$$M_O = \frac{S_O + a}{bS_O + c} \tag{8}$$

$$M_S = \frac{S_S + a}{bS_S + c} \tag{9}$$

$$M_L = \frac{S_L + a}{bS_L + c} \tag{10}$$

[1] August Ferdinand Moebius, 1790-1868, German mathematician and astronomer

These 3 equations for the 3 unknowns $\{a, b, c\}$ can easily be solved:

$$a = \frac{M_L(M_O - M_S)}{M_L(M_O + M_S) - 2M_O M_S} \tag{11}$$

$$b = \frac{2M_L - M_O - M_S}{M_L(M_O + M_S) - 2M_O M_S} \tag{12}$$

$$c = \frac{M_O - M_S}{M_L(M_O + M_S) - 2M_O M_S} \tag{13}$$

Thus, using equation 7 together with the now known parameters $\{a, b, c\}$, we can determine the reflection coefficient regardless of the details of the reflection bridge. Note, that also the output impedance of the reflection bridge Z_S is of no importance here.
How can we interpret the parameters $\{a, b, c\}$?
Equation 6 can be rewritten into the form

$$M = \frac{S + a}{c(\frac{b}{c}S + 1)} \tag{14}$$

The parameter a is closely related to what is called the directivity error [1]. If the gain of the reflect signal or the reference signal changes, only c will change, that is why c is called a tracking error [1]. To understand the meaning of the quotient b/c is not so simple. Let's assume that deep inside our reflection bridge there is an ideal reflection bridge with source impedance equal to the reference impedance Z_0, which is usually 50Ω. This impedance is transformed to the real port impedance Z_S by an error network described by an S-parameter matrix E_{ij}. This is shown in figure 2. The real

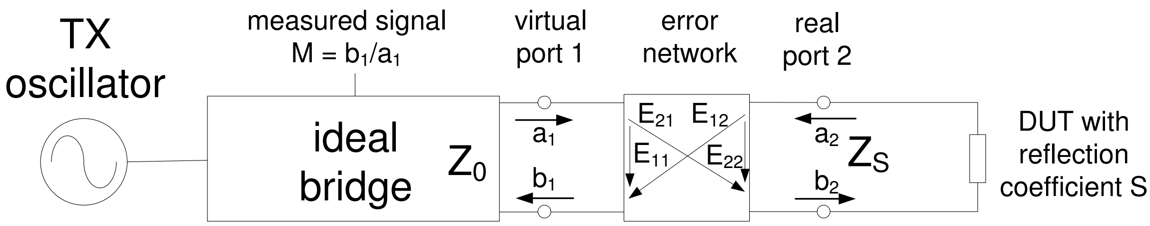

Figure 2: Real VNA bridge built up from an ideal VNA bridge and an error network.

bridge's output impedance Z_S corresponding to an output reflection coefficient

$$S_S = \left.\frac{b_2}{a_2}\right|_{a_1=0} = E_{22} \tag{15}$$

Now, what would our VNA measure due to the error network if port 2 was terminated with an impedance Z with corresponding reflection coefficient S? This is calculated in the Appendix A.1. Using equation 64 and the abbreviation $\Delta_E = E_{11}E_{22} - E_{12}E_{21}$ we obtain

$$M = \Gamma_{in} = \frac{b_1}{a_1} = \frac{\Delta_E S - E_{11}}{E_{22}S - 1} \tag{16}$$

Dividing enumerator and denominator by Δ_E we obtain

$$M = \frac{S - \frac{E_{11}}{\Delta_E}}{\frac{E_{22}}{\Delta_E} S - \frac{1}{\Delta_E}} \tag{17}$$

This is exactly of the same form as equation 6. By inspection we find

$$a = -\frac{E_{11}}{\Delta_E} \tag{18}$$

$$b = \frac{E_{22}}{\Delta_E} \tag{19}$$

$$c = -\frac{1}{\Delta_E} \tag{20}$$

Now we can calculate b/c:

$$\frac{b}{c} = \frac{\frac{E_{22}}{\Delta_E}}{-\frac{1}{\Delta_E}} = -E_{22} \tag{21}$$

And we finally find the real port 2 reflection coefficient S_S corresponding to the output impedance of the real bridge Z_S:

$$S_S = E_{22} = -\frac{b}{c} \tag{22}$$

2 Measuring 2-port S-parameters

2.1 Reflection measurement

As we have seen above, the bridge output impedance is of no importance when measuring 1-port reflection coefficients. This picture dramatically changes, when 2-port S-parameters are to be measured with the VNA depicted in figure 1. In the latter case, both impedances of the VNA, namely the bridge output impedance Z_S and the detector impedance Z_L, both terminating the DUT, will influence the measurement result M. This is quite obvious for the reflection measurements when looking at equation 61 from the appendix and replacing S by S_L. The input reflection coefficient of our DUT in forward direction f is

$$S_f = S_{11} + \frac{S_L S_{12} S_{21}}{1 - S_{22} S_L} \tag{23}$$

If the detector impedance Z_L is equal to the reference impedance Z_0, which means $S_L = 0$, equation 23 reduces to $S_f = S_{11}$. Note, that in the general case S_f depends on all four S-parameters of the DUT.
If we want to measure the output reflection coefficient in backward direction b by turning the DUT around, we have to exchange indices 1 and 2 in equation 23:

$$S_b = S_{22} + \frac{S_L S_{21} S_{12}}{1 - S_{11} S_L} \tag{24}$$

2.2 Transmission measurements

When doing a transmission measurement τ, one evaluates the quotient of the detector voltage u_{det} divided by the reference signal n. Here we assume u_{det} to be normalized with respect to the reference impedance Z_0 like the reference signal n and the reflect signal r. Thus, u_{det} can simply be written in terms of wave amplitudes

$$u_{det} = a_2 + b_2 = b_2 S_L + b_2 = b_2(1 + S_L) \tag{25}$$

Thus the forward transmission signal reads

$$\tau_f = \frac{u_{det}}{n} = \frac{b_2(1 + S_L)}{\alpha a_1 + \beta b_1} \tag{26}$$

Using equation 59, we can replace b_2:

$$\tau_f = \frac{(1 + S_L)S_{21}}{1 - S_{22}S_L} \cdot \frac{a_1}{\alpha a_1 + \beta b_1} \tag{27}$$

The second quotient can be simplified to

$$\frac{a_1}{\alpha a_1 + \beta b_1} = \frac{1}{\alpha + \beta \frac{b_1}{a_1}} = \frac{1}{\alpha + \beta S_f} \tag{28}$$

Next we want to normalize τ_f by a thru calibration measurement τ_{Thru}. An ideal thru calibration standard yields $S_{11} = S_{22} = 0$ and $S_{12} = S_{21} = 1$. Thus for the thru calibration measurement equation 27 becomes

$$\tau_{Thru} = (1 + S_L) \cdot \frac{a_T}{\alpha a_T + \beta b_T} = \frac{1 + S_L}{\alpha + \beta \frac{b_T}{a_T}} = \frac{1 + S_L}{\alpha + \beta S_L} \tag{29}$$

Note that the wave amplitudes depend on the DUT and are different in equation 29 from those in equation 27. Now we can normalize the forward transmission signal dividing it by the thru calibration measurement:

$$T_f = \frac{\tau_f}{\tau_{Thru}} = \frac{\frac{(1+S_L)S_{21}}{1-S_{22}S_L} \cdot \frac{1}{\alpha+\beta S_f}}{\frac{1+S_L}{\alpha+\beta S_L}} = \frac{S_{21}}{1 - S_{22}S_L} \cdot \frac{\alpha + \beta S_L}{\alpha + \beta S_f} = \frac{S_{21}}{1 - S_{22}S_L} \cdot \frac{1 + \frac{\beta}{\alpha} S_L}{1 + \frac{\beta}{\alpha} S_f} \tag{30}$$

Note, that $\beta/\alpha = b/c = -S_S$ as can be seen from equations 5, 6 and 22. Thus we find

$$T_f = \frac{S_{21}}{1 - S_{22}S_L} \cdot \frac{1 - S_S S_L}{1 - S_S S_f} \tag{31}$$

We can find the backward thru response from this by swapping indices 1 and 2 and replacing subscript f by b.

$$T_b = \frac{S_{12}}{1 - S_{11}S_L} \cdot \frac{1 - S_S S_L}{1 - S_S S_b} \tag{32}$$

Now, we have a set of 4 measurement results S_f, S_b, T_f and T_b (equations 23, 24, 31, 32), which all depend on all four S-parameters S_{11}, S_{12}, S_{21}, S_{22} to be determined. Note that all parameters are

known or can be measured, namely a, b, c and $S_S = -b/c$ are known from the reflect calibration and S_L can be measured with the reflect calibrated bridge during the thru calibration.
It remains to solve equations 23, 24, 31 and 32 for the unknown S-parameters S_{11}, S_{12}, S_{21}, S_{22}, so the VNA can actually calculate the S-parameters from the measurement results.

2.3 Calculating the S-parameters from the measurement results

The following normalization steps are not straight forward but were made to bring my equations to the same form as those published by Agilent [1]. In that course, my error terms can be compared to Agilent's notation.
First, the reflect measurements from equations 23 and 24 are rewritten. These reflection coefficients can be transformed to the uncalibrated measurement results M_F and M_B by virtue of equation 6

$$M_f = \frac{S_f + a}{bS_f + c} \tag{33}$$

$$M_b = \frac{S_b + a}{bS_b + c} \tag{34}$$

Now we perform a renormalization:

$$M_{11} = \frac{c^2 M_f - ac}{c - ab} = \frac{cS_f}{bS_f + c} \tag{35}$$

$$M_{22} = \frac{c^2 M_b - ac}{c - ab} = \frac{cS_b}{bS_b + c} \tag{36}$$

We also renormalize the thru measurements T_f and T_b:

$$M_{21} = \frac{T_f}{1 - S_S S_L} \tag{37}$$

$$M_{12} = \frac{T_b}{1 - S_S S_L} \tag{38}$$

As can easily be seen[2], solving these M_{ij} for the desired S-parameters yields

[2]Remark of the author: after several pages of tedious calculations

$$S_{11} = \frac{M_{11}(M_{22}S_S + 1) - M_{12}M_{21}S_L}{D} \tag{39}$$

$$S_{21} = \frac{M_{21}(1 - M_{22}(S_L - S_S))}{D} \tag{40}$$

$$S_{22} = \frac{M_{22}(M_{11}S_S + 1) - M_{12}M_{21}S_L}{D} \tag{41}$$

$$S_{12} = \frac{M_{12}(1 - M_{11}(S_L - S_S))}{D} \tag{42}$$

$$D = (S_S M_{11} + 1)(S_S M_{22} + 1) - S_L^2 M_{12} M_{21} \tag{43}$$

These equations are identical with those published in Agilent's paper [1] on page 20 if the identifications described in appendix A.2 are applied.
Now, just for completeness, we can replace the M_{ij}-terms through equations 35-38:

$$\Delta = b^2 S_L^2 (S_b S_f T_b T_f - 1) + bc S_L (S_b S_L T_b T_f + S_f S_L T_b T_f - 2) + c^2 (S_L^2 T_b T_f - 1) \tag{44}$$

$$S_{11} = \frac{b^2 S_f S_L (S_b T_b T_f - S_L) + bc S_L (S_b T_b T_f + S_f (T_b T_f - 2)) + c^2 (S_L T_b T_f - S_f)}{\Delta} \tag{45}$$

$$S_{21} = \frac{T_f (b S_f + c)(b S_L + c)(S_b S_L - 1)}{\Delta} \tag{46}$$

$$S_{22} = \frac{b^2 S_b S_L (S_f T_b T_f - S_L) + bc S_L (S_b (T_b T_f - 2) + S_f T_b T_f) + c^2 (S_L T_b T_f - S_b)}{\Delta} \tag{47}$$

$$S_{12} = \frac{T_b (b S_b + c)(b S_L + c)(S_f S_L - 1)}{\Delta} \tag{48}$$

Assuming that the detector has an ideal match, i.e. $S_L = 0$, these equations reduce to

$$S_{11} = S_f \tag{49}$$

$$\begin{aligned} S_{21} &= T_f(1 + \frac{b}{c} S_f) \\ &= T_f(1 - S_S S_f) \\ &= T_f \frac{a \cdot b - c}{c(b \cdot M_f - 1)} \end{aligned} \tag{50}$$

$$S_{22} = S_b \tag{51}$$

$$\begin{aligned} S_{12} &= T_b(1 + \frac{b}{c} S_b) \\ &= T_b(1 - S_S S_b) \\ &= T_b \frac{a \cdot b - c}{c(b \cdot M_b - 1)} \end{aligned} \tag{52}$$

Note that in the last four equations equations the forward and backward directions are decoupled. This might be a good alternative if only one signal direction can be measured and Z_L is close to Z_0. Generally, Z_L can be controlled more easily than Z_S. Also note, that these decoupled equations still compensate for a nonperfect bridge output impedance Z_S.
An even better approximation for the transmission correction is the so called "enhanced response calibration" (ERC) [3]. It can be derived from equation 31 by setting $S_{22} = 0$.

$$S_{21} = T_f \cdot \frac{1 - S_S S_{11}}{1 - S_S S_L} \tag{53}$$

By doing so, one suppresses multiple reflections between DUT output and detector input. This is a good approximation for reasonably well matched DUTs. For highly mismatched DUTs this can still be a good approximation, if the detector reflection coefficient S_L is close to zero as was shown in the previous approximation.

2.4 Isolation calibration

So far, we have only made use of 5 independent parameters in our calibration scheme, namely a, b, c, S_L and τ_{Thru} in forward direction. The same 5 parameters are recycled for the backward direction. So, we have used a total of calibration 10 parameters so far, which means 2 parameters are missing to a 12-term error correction. The two missing parameters are the instrument isolation in forward and backward direction. For the unidirectional VNA both are identical. As can be seen from Agilent's equations 74 and 76, the isolation measurement is simply subtracted from the thru measurement. This is only an approximation. It does not take into account that the isolation may depend on the terminations of the TX and RX ports. Still, it is useful if either the isolation is very good or if the dependencies on TX and RX port terminations are weak.

A Appendix

A.1 Relationship between input reflection coefficient and output termination of a 2-port

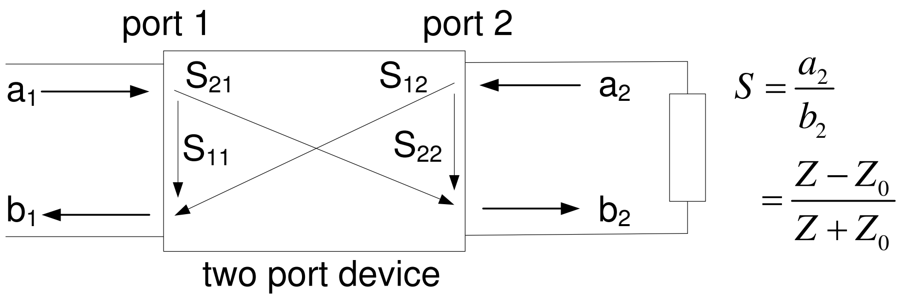

Figure 3: Two-port device terminated with impedance Z.

The following relationship hold between the wave amplitudes shown in figure 3:

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{54}$$

This can also be written as two scalar equations:

$$b_1 = S_{11}a_1 + S_{12}a_2 \tag{55}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \tag{56}$$

Since the wave b_2 is reflected by the termination impedance Z with corresponding reflection coefficient S, we can write:

$$S = \frac{a_2}{b_2} \tag{57}$$

or by virtue of equation 56

$$b_2 = S_{21}a_1 + S_{22}Sb_2 \tag{58}$$

Solving for b_2 we obtain

$$b_2 = \frac{S_{21}a_1}{1 - SS_{22}} \tag{59}$$

Now we can calculate the input reflection coefficient for port 1 from equation 55:

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}a_2}{a_1} \tag{60}$$

Inserting b_2 from 56 leads to

$$\Gamma_{in} = S_{11} + \frac{SS_{12}S_{21}}{1 - S_{22}S} \tag{61}$$

Bringing this to a common denominator

$$\Gamma_{in} = \frac{(S_{11}S_{22} - S_{12}S_{21})S - S_{11}}{S_{22}S - 1} \tag{62}$$

and using an abbreviation for the determinant of the S-matrix

$$\Delta_S = S_{11}S_{22} - S_{12}S_{21} \tag{63}$$

we obtain the final result

$$\Gamma_{in} = \frac{b_1}{a_1} = \frac{\Delta_S S - S_{11}}{S_{22}S - 1} \tag{64}$$

A.2 Identifications of Agilent's error terms

The error terms in [1] can be identified with ours.

A.2.1 Forward error terms

$$e_{11} = S_S = -\frac{b}{c} \tag{65}$$

$$e_{22} = S_L \tag{66}$$

$$e_{00} = \frac{a}{c} \tag{67}$$

$$e_{10}e_{01} = \frac{1}{c} - \frac{ab}{c^2} \tag{68}$$

Note that $e_{10}e_{32}$ is related to the thru calibration. It is effectively removed in our equations by working with the renormalized M_{ij}'s , see section A.2.3.

A.2.2 Backward error terms

$$e'_{11} = S_L \tag{69}$$

$$e'_{22} = S_S = -\frac{b}{c} \tag{70}$$

$$e'_{33} = \frac{a}{c} \tag{71}$$

$$e'_{23}e'_{32} = \frac{1}{c} - \frac{ab}{c^2} \tag{72}$$

Note that $e'_{23}e'_{01}$ is related to the thru calibration. It is effectively removed in our equations by working with the renormalized M_{ij}'s , see section A.2.3.

A.2.3 M_{ij}-Terms

$$\frac{S_{11M} - e_{00}}{e_{10}e_{01}} = M_{11} = \frac{c^2 M_f - ac}{c - ab} = \frac{cS_f}{bS_f + c} \tag{73}$$

$$\frac{S_{21M} - e_{30}}{e_{10}e_{32}} = M_{21} = \frac{T_f}{1 - S_S S_L} \tag{74}$$

$$\frac{S_{22M} - e'_{33}}{e'_{23}e'_{32}} = M_{22} = \frac{c^2 M_b - ac}{c - ab} = \frac{cS_b}{bS_b + c} \tag{75}$$

$$\frac{S_{12M} - e'_{03}}{e'_{23}e'_{01}} = M_{12} = \frac{T_b}{1 - S_S S_L} \tag{76}$$

A.3 Comparison of notations

The following translation table is useful when comparing literature results from various sources.
HP notation [2] vs. Agilent notation [1] vs. my notation

Forward direction:

$$E_{DF} = e_{00} = \frac{a}{c} \tag{77}$$

$$E_{SF} = e_{11} = S_S \tag{78}$$

$$E_{RF} = e_{10}e_{01} = \frac{1}{c} - \frac{ab}{c^2} \tag{79}$$

$$E_{LF} = e_{22} = S_L \tag{80}$$

$$E_{TF} = e_{10}e_{32} = \tau_{Thru} \cdot (1 - S_S S_L) \qquad \text{(thru cal measurement)} \tag{81}$$

$$E_{XF} = e_{30} \qquad \text{(isolation)} \tag{82}$$

$$S_{21M} = S_{21M} = \tau_f \tag{83}$$

$$S_{11M} = S_{11M} = M_f \tag{84}$$

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Backward direction:

$$E_{DR} = e'_{33} = \frac{a}{c} \tag{86}$$

$$E_{SR} = e'_{22} = S_S \tag{87}$$

$$E_{RR} = e'_{23}e'_{32} = \frac{1}{c} - \frac{ab}{c^2} \tag{88}$$

$$E_{LR} = e'_{11} = S_L \tag{89}$$

$$E_{TR} = e'_{23}e'_{01} = \tau_{Thru} \cdot (1 - S_S S_L) \qquad \text{(thru cal measurement)} \tag{90}$$

$$E_{XR} = e'_{03} \qquad \text{(isolation)} \tag{91}$$

$$S_{12M} = S_{12M} = \tau_b \tag{92}$$

$$S_{22M} = S_{22M} = M_b \tag{93}$$

(94)

References

[1] Doug Rytting, "Network Analyzer Error Models and Calibration Methods", Agilent.
The pdf document can be downloaded from:
http://cpd.ogi.edu/IEEE-MTT-ED/Network%20Analyzer%20Error%20Models%20and%20Calibration%20Methods.pdf

[2] User's Guide, HP 8753D Network Analyzer, Hewlett-Packard.
The pdf document can be downloaded from:
http://cp.literature.agilent.com/litweb/pdf/08753-90257.pdf

[3] Agilent application note AN 1287-3: "Applying Error Correction to Network Analyzer Measurements"
The pdf document can be downloaded from:
http://cp.literature.agilent.com/litweb/pdf/5965-7709E.pdf

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